Effective Chiral Lagrangian from Dual Resonance Models

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Abstract

Parameters of the effective chiral lagrangian (EChL) of orders $O(p^4)$ and $O(p^6)$ are extracted from low–energy behaviour of dual resonance models for $\pi\pi$ and πK scattering amplitudes. Dual resonance models are considered to be good candidates for the resonance spectrum and for hadronic scattering amplitudes in the large N_c limit of QCD. We discuss dual resonance models in the presence of spontaneous and explicit chiral symmetry breaking. Obtained parameters of the EChL are used to estimate chiral corrections up to the sixth order to various low–energy characteristics of $\pi\pi$ and πK scattering amplitudes.

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I. INTRODUCTION

The technique of the Effective Chiral Lagrangian (EChL) provides us with a systematic way of low-energy expansion of correlators of different colourless currents in Quantum Chromodynamics [1–3]. The information about large distance behaviour of the QCD is hidden in a finite set of coupling constants if we restrict ourselves to finite order in the momentum expansion. In the language of the EChL the expansion of the Green functions in external momenta and quark masses corresponds to an expansion in number of meson loops. With increasing order of the chiral expansion one has to extend the EChL by introducing couplings with increasing number of derivatives and increasing power of quark masses. Number of terms of the EChL grows rapidly with the expansion order. For example, in the leading order there are two possible chiral couplings, in the next-to-leading order Gasser and Leutwyler [4] determined ten low–energy constants which are not fixed by chiral symmetry requirements. At the next-to-next-to-leading $(O(p^6))$ order there are more than a hundred new low–energy constants [5]. An estimate of these sixth order constants is important to calculate the analytical (polynomial) part of the chiral sixth order corrections to the meson Green functions. Such kind of corrections arise from tree graphs of the sixth order EChL $(\mathcal{L}^{(6)})$, other contributions in this order of the chiral counting arise from one and two loops graphs of the fourth $(\mathcal{L}^{(4)})$ and second order EChL $(\mathcal{L}^{(2)})$ correspondingly. The loop corrections are expressed in terms of the known parameters of the fourth and second order EChL, whereas the polynomial corrections being technically easily calculable (tree graphs) depend on unknown constants. Hence to calculate the complete next-to-next-to-leading contributions to different meson Green functions one has to pin down the sixth order EChL coupling constants.

In this paper we determine some of the coupling constants of the sixth order EChL, in particular we calculate the polynomial part of the sixth order corrections to the elastic $\pi\pi$ and πK scattering amplitudes. To this end we use large N_c (number of colours) arguments or equivalently impose the Okubo–Zweig–Iizuki (OZI) rule. It is known that

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the fourth order constants determined in ref. [4] respect the OZI rule with good accuracy. Moreover in the large N_c limit the parameters of the effective chiral lagrangian can be related to the resonance spectrum [6–9] by contraction of the resonance contributions to the (pseudo)Goldstone scattering amplitudes. To ensure the chiral symmetry of resulting EChL one can either impose the chiral symmetry on the coupling of resonances to pions [6,7] or impose some relations on resonance spectrum [8,9] in the spirit of Weinberg's approach to the algebraic realization of the chiral symmetry [10,11]. The latter approach apart of predictions for the EChL parameters gives an infinite set of equations for resonance spectrum. These equations were derived and analyzed in ref. [9], it was shown there that the equations on the spectrum of the $\pi\pi$ resonances ensure the duality properties of the $\pi\pi$ scattering amplitude. Phenomenologically the duality of hadronic amplitudes was suggested in the sixties [12] as a certain relation between two ways of describing scattering amplitudes: the Regge pole exchange at high energies and resonance dominance at low energies. Later explicit models for hadron interaction implementing duality were constructed [13] and found to be in an agreement with experimental data [14]. Almost immediately it was found that the dual resonance amplitudes arise naturally in the quantum theory of the extended objects – strings. Now there is considerable theoretical belief that QCD in the large N_c limit corresponds to some string theory [15], though the particular form of the theory is not found. This task is difficult because it involves comparing field theory (QCD) in which we can not compute hadron amplitudes, with a string theory in which basically all one can do is to compute S- matrix in the narrow resonance approximation. Manifestations of possible underlying string dynamics in hadronic spectrum and interactions were recently discussed in refs. [21–23].

We make use of the dual resonance models (DRM) to estimate of the parameters of the sixth order chiral lagrangian. First, we study the conditions imposed by low–energy theorems on the dual resonance (string) models. Expand then the obtained amplitudes with " built in " soft–pion theorems at low energies and comparing the resulting expan-

sions with those given by the EChL we are able to fix the low–energy constants of the sixth order EChL. In principle, one can saturate sum rules relating the EChL and resonances spectrum derived in [6–9] by phenomenological resonance spectrum, unfortunately the corresponding sum rules for the sixth order EChL are very sensitive to experimental uncertainties, especially in the scalar channel where the spectroscopic data are controversial. Instead of that, we shall use the dual resonance models as models for resonance spectrum and their interactions. These models possess many attractive properties, in particular the dual resonace amplitudes have a correct Regge high energy behaviour and hence incorporate naturally the algebraic realization of chiral symmetry [10]. Also they predict a correct resonance mass spectrum.

The paper is organized as follows. In sect. II we introduce sixth order effective chiral lagrangian relevant for our purposes and fix our notations. In sect. III we discuss the polynomial part of the chiral corrections to the $\pi\pi$ and πK amplitudes in the next-to-next-to-leading order. In particular we give an explicit expression for the tree-level $\pi\pi$ and πK amplitudes in the sixth order in terms of the low–energy constants of the $\mathcal{L}^{(6)}$. Dual resonance $\pi\pi$ and πK amplitudes with spontaneously and explicitly broken chiral symmetry are constructed in sect. IV. We show that the soft–pion theorems impose very strong conditions on the dual amplitudes; that enables us in sect. V to calculate some of the low–energy constant of the sixth order EChL and compare them with chiral quark model predictions. Obtained parameters are used to estimate polynomial (analytical) part of the sixth order contribution to the low–energy scattering parameters (scattering lengths, slope parameters, etc.). Our summary and conclusions are surveyed in sect. VI.

II. EFFECTIVE CHIRAL LAGRANGIAN TO $O(P^6)$

In the lowest order of momentum expansion $O(p^2)$ the interactions of (pseudo)Goldstone mesons (pions, kaons and eta mesons) are described by the famous Weinberg lagrangian [1,2]:

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4} tr(L_\mu L_\mu) + \frac{F_0^2 B_0}{4} tr(\chi), \tag{2.1}$$

where $\chi = 2B_0(\hat{m}U + U^\dagger \hat{m})$, $L_\mu = iU\partial_\mu U^\dagger$, $\hat{m} = \mathrm{diag}(m, m, m_s)$ is a quark mass matrix and F_0 and B_0 are low-energy coupling constants carrying an information about long-distance behaviour of the QCD. The latter are related to pion decay constant and quark condensate in the chiral limit:

$$F_0 = \lim_{m_q \to 0} F_\pi,$$

$$B_0 = -\lim_{m_q \to 0} \frac{\langle \bar{\psi}\psi \rangle}{F_\pi}.$$

The chiral field $U(x)$ is a unitary 3×3 matrix and is parametrized in terms of eight pseudoscalar meson fields π, K and η:

$$U(x) = e^{i\Pi},$$

$$\Pi = \begin{pmatrix} \frac{\pi^0}{F_\pi} + \frac{\eta}{\sqrt{3}F_\eta} & -\sqrt{2}\frac{\pi^+}{F_\pi} & -\sqrt{2}\frac{K^+}{F_K} \\ -\sqrt{2}\frac{\pi^-}{F_\pi} & -\frac{\pi^0}{F_\pi} + \frac{\eta}{\sqrt{3}F_\eta} & -\sqrt{2}\frac{K^0}{F_K} \\ -\sqrt{2}\frac{K^-}{F_K} & -\sqrt{2}\frac{\bar{K}^0}{F_K} & -\frac{2\eta}{\sqrt{3}F_\eta} \end{pmatrix}, \tag{2.2}$$

with decay constants normalized as $F_\pi = 93.3$ MeV, $F_K \approx 1.2F_\pi$.

In the next $O(p^4)$ order the interactions of the (pseudo)Goldstone mesons are described by the following EChL (we write only terms surviving in the large N_c limit) [4][1]:

$$\mathcal{L}^{(4)} = (2L_2 + L_3) \; \mathrm{tr} \; (L_\mu L^\mu L_\nu L^\nu) + L_2 \; \mathrm{tr} \; (L_\mu L_\nu L^\mu L^\nu)$$

$$+ L_5 \; \mathrm{tr} \; (L_\mu L^\mu \chi) + L_8 \; \mathrm{tr} \; (\chi^2). \tag{2.3}$$

For the parameters of the fourth order EChL we use here notations of Gasser and Leutwyler [4]. We see that the fourth order EChL has in the large N_c limit four independent parameters.[2]

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[1]The fourth order EChL without symmetry breaking term containing derivatives was analyzed for the first time in ref. [16]

[2]Without taking the large N_c limit it depends on eight parameters [4]

All sixth order terms of the EChL were classified in ref. [5]. For our analysis of the analytical sixth order chiral contributions to the scattering amplitudes we need terms which are not vanishing in the leading order of the $1/N_c$ expansion and contribute to the pion–pion and pion–kaon scattering. This lagrangian has the form:

$$
\begin{aligned}
\mathcal{L}^{(6)} = {} & K_1 \text{ tr } (\partial_\sigma L_\mu \partial^\sigma L^\mu L_\nu L^\nu) + K_2 \text{ tr } (\partial_\sigma L_\mu \partial^\sigma L^\nu L^\mu L_\nu) \\
& + K_3 \text{ tr } (\partial_\sigma L_\mu \partial^\sigma L^\nu L_\nu L^\mu) + K_4 \text{ tr } (\partial_\sigma L_\mu L_\nu \partial^\sigma L^\mu L^\nu) \\
& + K_5 (\text{ tr } (\partial_\sigma L_\mu L^\mu \partial^\sigma L^\nu L_\nu) + \text{ tr } (\partial_\sigma L_\mu L_\nu \partial^\sigma L^\nu L^\mu)) \\
& + K_6 \text{ tr } (\chi L_\mu L^\mu L_\nu L^\nu) + K_7 \text{ tr } (\chi L_\mu L_\nu L^\mu L^\nu) + K_8 \text{ tr } (\chi L_\mu L_\nu L^\nu L^\mu) \\
& + K_9 \text{ tr } (L_\mu L^\mu \chi^2) + K_{10} \text{ tr } (L_\mu \chi L^\mu \chi) + K_{11} \text{ tr } (L_\mu \bar{\chi} L^\mu \bar{\chi}) \\
& + K_{12} \text{ tr } (L_\mu L^\mu \bar{\chi} \bar{\chi}) + K_{13} \text{ tr } (\chi \bar{\chi}^2) + K_{14} \text{ tr } (\chi^3),
\end{aligned} \tag{2.4}
$$

where $\bar{\chi} = 2B_0(\hat{m}U - U^\dagger \hat{m})$ and χ, $\hat{m}$ and L_μ are defined after eq.(2.1). New coupling constants $K_{1...14}$ determine the polynomial (analytical) part of low-energy behaviour of the two- and four-point meson Green functions in the large N_c limit (OZI rule). Non–analytical part of the Green functions and the violation of the OZI rule arise from mesonic loops. The non–analytic contributions to the $\pi\pi$ and πK scattering amplitudes to one loop were calculated in refs. [3,17], part of two loops contributions to $\pi\pi$ scattering were calculated recently in refs. [18], the low energy $\pi\pi$ amplitude to one and two loops were recently obtained in generalized chiral perturbation theory [19], the complete two–loop calculations in standard chiral perturbation theory will be finished soon [20]. To calculate the $O(p^6)$ corrections to the low–energy scattering amplitudes completely one needs (along with two loops contributions) to know the parameters $K_{1...14}$ of the sixth order EChL. Apart from " practical" value, the determination of the sixth order EChL parameters $K_{1...14}$ has, we think, a wider theoretical significance since these constants are related to fine features of the spontaneous chiral symmetry breaking in QCD and they can be used to check predictions of various models for chiral symmetry breaking in QCD.

III. LOW-ENERGY $\pi\pi$ AND πK SCATTERING AMPLITUDES

In this section we calculate the polynomial part of the chiral corrections to masses, decay constants and $\pi\pi$ and πK scattering amplitudes in terms of the parameters of the sixth order EChL given by eqs. (2.1,2.3,2.4). We follow closely the technique described in refs. [3,4,17], hence we give below only the results of our calculations without technical details.

A. Masses and decay constants

Masses and decay constants of pions and kaons are extracted from two point correlation function of axial currents. The result of calculations for polynomial part of the chiral corrections is:

$$M_\pi^2 = 2mB_0\{1 + m\frac{16B_0}{F_0^2}(2L_8 - L_5) - m^2\frac{236B_0^2}{F_0^4}L_5(2L_8 - L_5)$$
$$+ m^2\frac{64B_0^2}{F_0^2}(K_9 + K_{10} + 2K_{13} + 3K_{14}) + O(m^3)\}, \tag{3.1}$$

$$M_K^2 = (m + m_s)B_0\{1 + (m + m_s)\frac{8B_0}{F_0^2}(2L_8 - L_5) - (m + m_s)^2\frac{64B_0^2}{F_0^4}L_5(2L_8 - L_5)$$
$$+ \frac{32B_0^2}{F_0^2}((m^2 + m_s^2)(K_9 + K_{13} + 3K_{14}) + mm_s(2K_{10} + 2K_{13}))\}, \tag{3.2}$$

$$F_\pi^2 = F_0^2\{1 + m\frac{16B_0}{F_0^2}L_5 - \frac{64m^2B_0^2}{F_0^2}(K_9 + K_{10}) + O(m^3)\}, \tag{3.3}$$

$$F_K^2 = F_0^2\{1 + (m + m_s)\frac{8B_0}{F_0^2}L_5 - \frac{32B_0^2}{F_0^2}((m^2 + m_s^2)K_9 + 2mm_sK_{10})\}, \tag{3.4}$$

B. Scattering amplitudes

Let us consider the elastic $\pi\pi$–scattering process

$$\pi_a(k_1) + \pi_b(k_2) \rightarrow \pi_c(k_3) + \pi_d(k_4).$$

($a, b, c, d = 1, 2, 3$ are the isotopic indices and $k_1, .., k_4$ — pion momenta.) Its amplitude M^{abcd} can be written in the form:

$$M^{abcd} = \delta^{ab}\delta^{cd}A + \delta^{ac}\delta^{bd}B + \delta^{ad}\delta^{bc}C, \tag{3.5}$$

where A, B, C are the scalar functions of Mandelstam variables s, t, u:

$$s = (k_1 + k_2)^2, \qquad t = (k_1 - k_3)^2, \qquad u = (k_1 - k_4)^2, \tag{3.6}$$

obeying the Bose–symmetry requirements:

$$A(s, t, u) = A(s, u, t) \ ,$$
$$B(s, t, u) = A(t, s, u) \ , \tag{3.7}$$
$$C(s, t, u) = A(u, t, s) \ .$$

The amplitude of the πK scattering process

$$\pi_a(k_1) + K_\alpha(k_2) \to \pi_b(k_3) + K_\beta(k_4). \tag{3.8}$$

can be expressed in terms of two (iso)scalar functions $T_+(\nu, t)$ and $T_-(\nu, t)$ by

$$M^{ab}_{\alpha\beta} = \delta^{ab}\delta_{\alpha\beta}T_+(\nu, t) + i\epsilon^{abc}\sigma^c_{\beta\alpha}T_-(\nu, t), \tag{3.9}$$

where invariant variable $\nu = s - u$ is expressed via Mandelstam variables eq. (3.6). At low momenta one can expand the (iso)scalar amplitudes, $T_+(\nu, t)$ and $T_-(\nu, t)$ in power series of invariant kinemantical variables:

$$A(s, t) = \sum_{i,j} A_{ij}(m)s^i t^j, \tag{3.10}$$

$$T_+(\nu, t) = \sum_{k,l} t^+_{2k\, l}(m_s)\nu^{2k}t^l, \tag{3.11}$$

$$T_-(\nu, t) = \sum_{k,l} t^-_{2k+1\, l}(m_s)\nu^{2k+1}t^l. \tag{3.12}$$

Non-analytical parts of the amplitudes (like $E^4 log(E)$) are suppressed by additional factors of $1/N_c$. Parameters of the near threshold expansion depend on quark masses. From

the Effective Chiral Lagrangian, eqs. (2.1,2.3,2.4), one gets an expression for the low-energy parameters of the $\pi\pi$ and πK scattering amplitudes as series in quark mass:

$\underline{\pi\pi\ parameters}$[3]

$$A_{00} = -\frac{2mB_0}{F_0^2} + \frac{64m^2B_0^2}{F_0^4}(3L_2 + L_3)$$
$$+ \frac{64m^3B_0^3}{F_0^4}\Big\{2(K_1 + K_2 + K_3 - K_4 - 2K_5)$$
$$+ (\tilde{K}_6 + \tilde{K}_7 + \tilde{K}_8 + 16(3L_2 + L_3)(2L_8 - L_5))$$
$$+ (4K_9 + 4K_{10} - 8K_{11} + 8K_{12} + 8K_{13} + 6K_{14} + 64L_5(3L_5 - 4L_8))\Big\}, \tag{3.13}$$

$$A_{01} = -\frac{64mB_0L_2}{F_0^4} - \frac{64m^2B_0^2}{F_0^4}\Big\{(K_2 + K_3 - 2K_5) + 2\tilde{K}_7 + 16L_2(2L_8 - L_5)\Big\}, \tag{3.14}$$

$$A_{02} = \frac{8L_2}{F_0^4} + \frac{8mB_0}{F_0^4}\Big\{(K_2 + K_3 - 2K_5) + 2\tilde{K}_7\Big\}, \tag{3.15}$$

$$A_{03} = 0, \tag{3.16}$$

$$A_{10} = \frac{1}{F_0^2} - \frac{32mB_0}{F_0^4}(2L_2 + L_3)$$
$$+ \frac{32m^2B_0^2}{F_0^4}\Big\{(-3K_1 - 3K_2 - K_3 + 6K_4 + 4K_5)$$
$$- 2(\tilde{K}_6 + \tilde{K}_8) - 16(2L_2 + L_3)(2L_8 - L_5) - 2(K_9 + K_{10} - 4K_{11} + K_{12})\Big\}, \tag{3.17}$$

$$A_{11} = \frac{8L_2}{F_0^4} + \frac{16mB_0}{F_0^4}\Big\{2K_2 + K_3 - 3K_4 - 2K_5 + \tilde{K}_7\Big\}, \tag{3.18}$$

$$A_{12} = A_{21} = \frac{1}{F_0^4}(-3K_2 - K_3 + 6K_4 + 2K_5), \tag{3.19}$$

$$A_{20} = \frac{4(2L_2 + L_3)}{F_0^4} + \frac{8mB_0}{F_0^4}\Big\{(3K_1 + 3K_2 - 6K_4 - 4K_5) + \tilde{K}_6 + \tilde{K}_8\Big\}, \tag{3.20}$$

$$A_{30} = -\frac{2}{F_0^4}(K_1 + K_2 - 2K_4 - 2K_5), \tag{3.21}$$

where we introduce the following notations:

$$\tilde{K}_6 = K_6 - \frac{8}{F_0^2}(2L_2 + L_3)L_5,$$
$$\tilde{K}_7 = K_7 - \frac{16}{F_0^2}L_2L_5,$$

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[3]Corresponding expressions for the $\pi\pi$ scattering lengths and slope parameters are given in Appendix A.

$$\tilde{K}_8 = K_8 - \frac{8}{F_0^2}(2L_2 + L_3)L_5.$$

The result for the low–energy parameters of the πK scattering amplitude (with $m_u = m_d = 0$) is the following:

$\underline{\pi K \ parameters}$

$$t_{00}^+ = 0 \qquad (\text{ exactly}) \tag{3.22}$$

$$t_{01}^+ = \frac{1}{4F_0^2} - \frac{4m_s B_0}{F_0^4}(2L_2 + L_3) \tag{3.23}$$

$$+ \frac{m_s^2 B_0^2}{F_0^4}\left\{-2K_1 - 4\tilde{K}_8 + 8(K_9 - K_{12}) - 64(2L_2 + L_3)(2L_8 - L_5)\right\} \tag{3.24}$$

$$t_{02}^+ = \frac{12L_2 + 5L_3}{2F_0^4} + \frac{m_s B_0}{4F_0^4}\left\{(K_2 - 3K_3 - 8K_5) + 2(\tilde{K}_6 + \tilde{K}_7 + 4\tilde{K}_8)\right\} \tag{3.25}$$

$$t_{03}^+ = \frac{1}{8F_0^4}(-7K_1 - K_2 + 2K_3 + 2K_4 + 10K_5) \tag{3.26}$$

$$t_{20}^+ = \frac{4L_2 + L_3}{2F_0^4} + \frac{m_s B_0}{4F_0^4}\left\{(K_2 + K_3) + 2(\tilde{K}_6 + \tilde{K}_7)\right\} \tag{3.27}$$

$$t_{21}^+ = \frac{1}{8F_0^4}(3K_1 - 3K_2 - 2K_3 + 6K_4 - 2K_5) \tag{3.28}$$

$$t_{10}^- = \frac{1}{4F_0^2} \qquad (\text{exactly!}) \tag{3.29}$$

$$t_{11}^- = \frac{-L_3}{F_0^4} + \frac{m_s B_0}{2F_0^4}\left\{(K_2 + K_3 - 4K_5) + 2(\tilde{K}_7 - \tilde{K}_6)\right\} \tag{3.30}$$

$$t_{12}^- = 3t_{30}^- = \frac{3}{8F_0^4}(-K_1 - K_2 + 2K_3 + 2K_4). \tag{3.31}$$

Let us stress that the parameters of the πK scattering amplitude t_{00}^+ and t_{10}^- given by eqs.(3.22,3.29) have no corrections due to non–zero strange quark mass in any order of m_s expansion, though there are corrections of order $O(m \cdot m_s)$ and $O(m_s/N_c)$. The former corrections are expected to be very small, whereas the latter appears due to loop contributions and are not considered in the present paper since we caclulate only polynomial part of the chiral corrections to the scattering amplitudes. These <u>exact</u> on-shell low energy theorems will enable us to fix parameters of the dual resonance models for $\pi\pi$ and πK scattering amplitudes.

IV. DUAL RESONANCE MODELS FOR SCATTERING OF THE (PSEUDO) GOLDSTONE PARTICLES

The dual resonance models were invented in 60's to describe some striking features of hadron interactions. To good accuracy the mesons and baryons lie on linear Regge trajectories, a wealth of high energy scattering data is modelled very well by single reggeon exchange (for a review see ref. [28]). Later it has been found that the dual resonance models follow from string theory and that duality is a consequence of the infinite-dimensional conformal symmetry of string theories. After the advent of the Quantum Chromodynamics much evidences have been found that QCD in large N_c limit might be equivalent to some string theory. Let us just list them:

- The success of Regge phenomenology [28]

- The perturbation expansion in the large N_c limit of QCD can be written as a sum over surfaces which may correspond to a sum over string world sheets [29]

- The strong coupling expansion for lattice gauge theory strongly resembles a string theory [33]

- The Wilson loop expectation values in the large N_c limit satisfy equations which is equivalent to those for one or another specific string theory strings [32]

- 2D QCD can be rewritten as a string theory [34]

Unfortunately, the precise form of the string theory corresponding to QCD is unknown. In the present paper we exploit well established facts about QCD to find conditions which are imposed on string theories (dual resonance models) by these facts. One of the most prominent phenomena occurring in QCD is the spontaneous breaking of chiral symmetry. It leads to numerous low–energy theorems for scattering amplitudes of the (pseudo)Goldstone particles (π, K and η), which are written compactly in terms of the effective chiral lagrangian, eqs.(2.1,2.3,2.4).

A dual resonance model for the $\pi\pi$ scattering amplitude consistent with low-energy theorems in the chiral limit and ghost free has been suggested in [30,31]. It has a form (in the chiral limit):

$$M^{abcd} = \text{ tr } (\tau^a\tau^b\tau^c\tau^d)V(s,t) + \text{non-cyclic permutations,}$$
$$V(s,t) = \lambda\frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t)))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))}, \tag{4.1}$$

where the ρ-meson Regge trajectory and the constant λ are chosen to be

$$\alpha_\rho(s) = \frac{1}{2} + \frac{s}{2m_\rho},$$
$$\lambda = -\frac{m_\rho^2}{\pi F_0^2}, \tag{4.2}$$

in order to ensure the low-energy theorem for the amplitude:

$$\lim_{s,t\to 0} A(s,t) = \frac{s}{F_0^2} + O(p^4). \tag{4.3}$$

The ampltude (4.1), besides correct low–energy properties, satisfies a Regge asymptotic restrictions at high energies. Moreover, the positions and residues of the resonance poles are in a good agreement with phenomenological ones. Hence the basic phenomenological features of the hadron interactions are implemented by a simple dual amplitude (4.1).

Performing low–energy expansion of the amplitude (4.1) one can immediately extract parameters L_1, L_2 and L_3 of the fourth order EChL eq.(2.3):

$$L_2 = 2L_1, \tag{4.4}$$

$$L_3 = -2L_2 \tag{4.5}$$

$$L_2 = \frac{F_0^2}{8m_\rho^2}\ln(2) \approx 1.25 \times 10^{-3}. \tag{4.6}$$

The first relation eq. (4.4) is identical to one following from the large N_c conditions for the meson scattering amplitude [4]. These conditions are "built in" in the dual resonance models through the Chan–Paton isotopic factor. The second relation, eq. (4.5), is exactly the relation predicted by integration of the non-topological chiral anomaly [24,26,25,27]

and it holds with any type of satellites added to a simple Lovelace–Shapiro amplitude (4.1) [9]. Moreover the numericl value of L_2 (4.6) is close to that found by Gasser and Leutwyler in ref. [4] $L_2 = (1.7 \pm 0.7) \cdot 10^{-3}$, to recent determination of this constant from analysis of the K_{l4} decay [36] $L_2 = (1.35 \pm 0.3) \cdot 10^{-3}$ and simultaneously to that obtained by integration of the non–topological chiral anomaly $L_2 = 1.58 \cdot 10^{-3}$. The values of the combination $2L_2 + L_3$ obtained in refs. [4,36] are consistent with zero[4].

To estimate other parameters entering eqs.(2.3,2.4) one has to introduce an explicit chiral symmetry breaking to the dual resonance model. In a pioneering works of Lovelace [30] and Shapiro [31] it was achieved by shifting the intercept of the ρ-meson Regge trajectory from $\frac{1}{2}$ to $\frac{1}{2} - M_\pi^2/2m_\rho^2$ to reproduce the Adler zero. Unfortunately, the Adler condition is an *off mass shell* one, whereas the dual (string) amplitudes can be defined and constructed consistently only *on mass shell* , and a continuation of those to unphysical region is ambiguous. Here we shall use a new way of introducing quark masses (explicit chiral symmetry breaking) into the dual model. Instead of using the Adler condition we shall impose *on mass shell* low energy theorems (like those given by eqs. (3.22,3.29)) on the dual amplitudes for the $\pi\pi$ and πK scatterings.

A. Dual resonance model for the $\pi\pi$ and πK scattering amplitudes. Non-zero quark masses

Now we generalize the dual amplitude for pions (4.1) for a case of small non-zero quark masses, in this case one can write generically:

$$V(s,t) = -\frac{m_\rho^2}{\pi F_0^2}(1 + a_1 m + a_2 m^2 + \ldots)\left\{\frac{\Gamma(1 - \alpha_\rho(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(s) - \alpha_\rho(t))}\right.$$

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[4]For the direct check of the relation $2L_2 + L_3 = 0$ dictated by non-topological chiral anomaly of QCD and dual (string) models one need to repeat the fitting procedure of ref. [36] using, among others, variable $2L_2 + L_3$

$$+ (b_1 m + b_2 m^2 + \ldots) \frac{\Gamma(1 - \alpha_\rho(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(2 - \alpha_\rho(s) - \alpha_\rho(t))} \Big\}, \tag{4.7}$$

where the intercept of the ρ-meson Regge trajectory has also mass corrections:

$$\alpha_\rho(s) = \frac{1}{2}(1 + i_1 m + i_2 m^2 + \ldots) + \frac{s}{2m_\rho^2}, \tag{4.8}$$

here m_ρ is a mass of the ρ meson in the chiral limit and coefficients i_k describes corrections to the intercept of the ρ meson trajectory and simultaneously the quark mass corrections to the m_ρ. We do not include corrections to the slope of the trajectory because they can be absorbed by a redifinition of the ρ mass.

It is easy to see that in the chiral limit the amplitude, eq. (4.7), coincides with (4.1). For the second term in eq. (4.7) we choose the simplest possible satellite term having no poles at zero momenta. The generalization for arbitrary satellites is straightforward but unnecessary for our purposes. Some of the unknown parameters a_i and b_i can be fixed by the low-energy theorems, eqs. (3.13, 3.17); the resulting amplitude has the form:

$$\begin{aligned}
V(s,t) = &-\frac{m_\rho^2}{\pi F_0^2}(1 - \frac{4mB_0 \ln 2}{m_\rho^2}(1 + \frac{(2\alpha_\rho(0) - 1)m_\rho^2}{M_\pi^2}) + a_2 m^2 + \ldots) \\
&\times \Big\{ \frac{\Gamma(1 - \alpha_\rho(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(s) - \alpha_\rho(t))} \\
&+ (\frac{2mB_0}{m_\rho^2} + (2\alpha_\rho(0) - 1) + b_3 m^3 + O(m^4)) \frac{\Gamma(1 - \alpha_\rho(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(2 - \alpha_\rho(s) - \alpha_\rho(t))} \Big\}. \tag{4.9}
\end{aligned}$$

The mass corrections to the intercept $\alpha_\rho(0)$ and parameters a_2 and b_3 are not fixed by the on–mass–shell low-energy theorems. The *off mass shell* Adler conditions being imposed on the amplitude eq. (4.9) give the following relation [30,35]:

$$(2\alpha_\rho(0) - 1) = i_1 m + i_2 m^2 + \ldots = -\frac{M_\pi^2}{m_\rho^2}. \tag{4.10}$$

We shall keep the parameters i_k (or, equivalently, $\alpha_\rho(0)$) free, because to implement the Adler conditions one has to know the continuations of the dual amplitude off mass shell. The latter problem is not solved in the dual (string) models.

Now let us construct a dual amplitude for the πK elastic scattering for the simplified case of $m_u = m_d = 0$ and $m_s \neq 0$. In this case we have very powerful low–energy theorems (3.22,3.29) which, as we shall see, fix parameters of the dual amplitude completely.

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The dual πK scattering amplitude depends on the ρ-meson trajectory (4.8) and K^* one, the latter generically has the form (we assume that $m_u = m_d = 0$):

$$\alpha_{K^*}(s) = \frac{1}{2}(1 + j_1 m_s + j_2 m_s^2 + \ldots) + \frac{s}{2m_\rho^2}(1 + n_1 m_s + n_2 m_s^2 + \ldots), \qquad (4.11)$$

where we introduce, on general grounds, the quark mass corrections to the intercept and the slope of the K^*-meson trajectory. The K^* meson mass (in large N_c limit) is determined by the equation:

$$\alpha_{K^*}(m_{K^*}^2) = 1. \qquad (4.12)$$

The dual resonance amplitude for the πK elastic scattering satisfying all the on mass shell low–energy theorems has a form:

$$T_{\pm}(\nu, t) = V_{\rho K^*}(s, t) \pm V_{\rho K^*}(u, t),$$

$$V_{\rho K^*}(s,t) = -\frac{m_\rho^2}{2\sqrt{\pi}F_0^2}\frac{\Gamma(1 + \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{2m_\rho^2})}{\Gamma(\frac{1}{2} + \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{2m_\rho^2})}\left\{\frac{\Gamma(1 - \alpha_{K^*}(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_{K^*}(s) - \alpha_\rho(t))}\right.$$

$$\left. - \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{2m_\rho^2}\frac{\Gamma(1 - \alpha_{K^*}(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(2 - \alpha_{K^*}(s) - \alpha_\rho(t))}\right\}, \qquad (4.13)$$

where we take into account the fact that the low–energy theorems can be satisfied only with the following relations between parameters of the $\pi\pi$ and πK dual amplitudes:

$$i_1 = 2j_1,$$

$$n_k = 0 \qquad \text{universality of the Regge trajectories slope !.} \qquad (4.14)$$

It is remarkable that soft pion theorems lead to the universality of the Regge trajectories slopes and also give a mass relation

$$\frac{m_{K^*}^2 - m_\rho^2}{m_\rho^2} = -(2\alpha_\rho(0) - 1)\frac{m_s}{2m} \approx -\frac{i_1 m_s}{2}. \qquad (4.15)$$

From this mass formula and Adler relation eq. (4.10) one gets the famous mass relation of Lovelace [30] :

$$m_{K^*}^2 = m_\rho^2 + M_\pi^2. \qquad (4.16)$$

By virtue of eq. (4.15) we shall use the mass difference $M_{K^*}^2 - m_\rho^2$ as a free parameter equivalent to $\alpha_\rho(0)$.

V. LOW–ENERGY EXPANSION OF THE DUAL RESONANCE AMPLITUDE

Expanding the dual resonance amplitudes, eqs. (4.9, 4.13), for $\pi\pi$ and πK scatterings at low momenta one can obtain some of the EChL parameters. In order to fix the others one needs to know additionally dual n-point amplitudes for pions. Unfortunately, less is known about n-point generalization of the Lovelace formula (4.1)[5]. Also the dual amplitudes in question depend on parameter(s) $\alpha_\rho(0)$ (or, equivalently, i_k) which are not fixed by soft pion theorems. We choose $\alpha_\rho(0)$ corresponding to the experimental values of the vector meson masses in eq. (4.15).

Expanding the dual resonance amplitudes, eqs.(4.9, 4.13), at low energies and comparing the result with eqs. (3.13-3.21) and eqs. (3.22-3.31) respectively one can fix the following parameters of the sixth order EChL:

$$
\begin{aligned}
K_1 &= \quad 0, \\
K_2 &= -\frac{F_0^2(\pi^2 + 15\ln^2 2)}{60 m_\rho^4}, \\
K_3 &= \quad \frac{F_0^2 \pi^2}{80 m_\rho^4}, \\
K_4 &= -\frac{F_0^2 \pi^2}{96 m_\rho^4}, \\
K_5 &= \quad \frac{F_0^2 \pi^2}{80 m_\rho^4}, \\
\tilde{K}_6 &= -\frac{F_0^2 \pi^2}{80 m_\rho^4} - \frac{F_0^2 \pi^2}{64 m_\rho^4}\frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2}, \\
\tilde{K}_7 &= \quad \frac{F_0^2(7\pi^2 + 60\ln^2 2)}{480 m_\rho^4} - \frac{F_0^2 \pi^2}{192 m_\rho^4}\frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2}, \\
\tilde{K}_8 &= \quad \frac{3F_0^2 \pi^2}{160 m_\rho^4} + \frac{F_0^2 \pi^2}{64 m_\rho^4}\frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2}, \\
K_9 - K_{12} &= \quad \frac{3F_0^2 \pi^2}{320 m_\rho^4} + \frac{F_0^2 \pi^2}{128 m_\rho^4}\frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2},
\end{aligned}
\tag{5.1}
$$

---

[5] See, though, a recent paper [37] where the n-point generalization of the Lovelace–Shapiro amplitude were suggested

$$K_{10} - K_9 = -\frac{F_0^2 \pi^2}{128 m_\rho^4} - \frac{F_0^2 \pi^2}{128 m_\rho^4} \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2},$$

This is the main result of the paper. Numerically from these equations one has (taking $m_{K^*} = 892$ MeV):

$$\begin{aligned}
K_1 m_\rho^2 &= \ \ 0, \\
K_2 m_\rho^2 &\approx -3.72 \cdot 10^{-3}, \\
K_3 m_\rho^2 &\approx \ \ 1.61 \cdot 10^{-3}, \\
K_4 m_\rho^2 &\approx -1.34 \cdot 10^{-3}, \\
K_5 m_\rho^2 &\approx \ \ 1.61 \cdot 10^{-3}, \\
\tilde{K}_6 m_\rho^2 &\approx -1.26 \cdot 10^{-3}, \\
\tilde{K}_7 m_\rho^2 &\approx \ \ 2.78 \cdot 10^{-3}, \\
\tilde{K}_8 m_\rho^2 &\approx \ \ 2.07 \cdot 10^{-3}, \\
(K_9 - K_{12}) m_\rho^2 &\approx \ \ 1.03 \cdot 10^{-3}, \\
(K_{10} - K_9) m_\rho^2 &\approx -0.82 \cdot 10^{-3},
\end{aligned} \tag{5.2}$$

In these numerical estimates for the parameters mB_0, F_0 and $2L_8 - L_5$ we use the values given by Gasser and Leutwyler [3,4]:

$$F_0 = 88 \text{MeV}, \tag{5.3}$$

$$2mB_0 = (141)^2 \text{MeV}^2, \tag{5.4}$$

$$m_s B_0 = (505)^2 \text{MeV}^2, \tag{5.5}$$

$$2L_8 - L_5 = (0 \pm 1.1) 10^{-3}. \tag{5.6}$$

In order to fix other coefficients we have to analyze not only scattering amplitudes but also mass and decay constants splittings. This can be done consistently only if two loop contributions to these quantities are taken into account, so this deserves further study. To estimate the polynomial contributions of the sixth order to $S-$ and $P-$wave $\pi\pi$ scattering lengths one needs to pin down the following combination of the parameters

$-K_9 - K_{10} + 4K_{11} - 2K_{12}$, this is equivalent to fixing parameters a_2 and b_3 in the dual resonance $\pi\pi$ scattering amplitude (4.9). From eqs. (5.2) one can assume tentatively that $|K_{9 \div 14}| \sim 10^{-3}/m_\rho^2$. We shall use this tentative numbers to estimate uncertainties due to unknown parameters.

A. Comparison with Chiral Quark Model

In the previous section we showed that the low–energy constants of the *fourth* order EChL obtained from the dual resonance models in the chiral limit coincide with corresponding constants obtained by integration of the non–topological chiral anomaly [24,26,25] and from the gradient expansion of the fermion determinant in the effective chiral quark model [27]. Let us compare the low–energy constants (5.2) with the corresponding constants obtained by gradient expansion of the fermion determinant in the effective chiral quark model.

According to Manohar and Georgi [38] we can describe the strong interactions at energies below the scale of chiral symmetry breaking by a set of fields consisting of $SU(N_f)_V$ multiplet of quarks with a dynamical mass M and Goldstone bosons. This picture of the low–energy QCD emerges naturally in the low–momenta limit from the instanton picture of QCD. According to ref. [27] the contents of QCD at low–momenta comes to dynamically massive quarks interacting with pseudoscalar fields whose kinetic energy appears only dynamically through quark loops. The basic quantities of the model, viz. the momentum-dependent quark mass $M(p)$ and the intrinsic ultra-violet cut-off have been also estimated in ref. [27] through the Λ_{QCD} parameter.

The low-momenta QCD partition function is given by the functional integral over pseudoscalar and quark fields (in the chiral limit):

$$\mathcal{Z} = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{D}\pi^A \, \exp\left(i \int d^4x \bar{\Psi} i D \Psi \right) \tag{5.7}$$

$$= \int \mathcal{D}\pi^A \, exp\left(i S_{eff}[\pi] \right), \tag{5.8}$$

$$S_{eff}[\pi] \;=\; -\mathrm{Sp}\log iD, \tag{5.9}$$

where iD denotes the Dirac differential operator entering the effective fermion action:

$$S_{eff}^{ferm} = \int d^4x\,\bar{\Psi}iD\Psi, \tag{5.10}$$

$$iD \;=\; (-i\slashed{\partial} + MU^{\gamma_5}), \tag{5.11}$$

with the pseudoscalar chiral field

$$U^{\gamma_5} = e^{i\pi^A\lambda^A\gamma_5}. \tag{5.12}$$

λ^A are Gell-Mann matrices and M is the dynamical quark mass which arises as a result of the spontaneous chiral symmetry breaking and is momentum-dependent. The momentum dependence of M introduces the natural ultra-violet cut-off for the theory given by eq. (5.8). Performing the expansion of the effective action for pions, given by the fermion determinant (5.9), in powers of pion momenta one reveals the EChL for pions in the large N_c limit. For the fourth order EChL this gives [27]:

$$L_2 = \frac{1}{12}\frac{N_c}{24\pi^2} \approx 1.58 \cdot 10^{-3},$$
$$L_3 = -2L_2.$$

In refs. [39,40] the low–energy constants of the sixth order EChL were calculated as functions of parameters of the effective chiral quark model, i.e. the constituent quark mass M and the effective cut–off Λ (proper–time regularization scheme were used):

$$K_1 = -\frac{3}{10}\frac{N_c}{96\pi^2 M^2}\Gamma(3,\frac{M^2}{\Lambda^2}),$$
$$K_2 = \frac{3}{10}\frac{N_c}{96\pi^2 M^2}\Gamma(3,\frac{M^2}{\Lambda^2}),$$
$$K_3 = \frac{N_c}{96\pi^2 M^2}[\frac{1}{5}\Gamma(2,\frac{M^2}{\Lambda^2}) + \frac{3}{40}\Gamma(3,\frac{M^2}{\Lambda^2})],$$
$$K_4 = \frac{1}{5}\frac{N_c}{96\pi^2 M^2}\Gamma(3,\frac{M^2}{\Lambda^2}),$$
$$K_5 = \frac{N_c}{96\pi^2 M^2}\cdot[\frac{1}{10}\Gamma(2,\frac{M^2}{\Lambda^2}) - \frac{3}{80}\Gamma(3,\frac{M^2}{\Lambda^2})],$$

where $\Gamma(n, x)$ is the incomplete gamma function. We see that it is impossible to reproduce our values for the corresponding low–energy constants (5.2), adjusting two parameters of the effective chiral quark model . In our view this discrepancy might be due to neglection of nonlocality of the corresponding effective fermion action (5.10) (say, owing to momentum dependence of the constituent quark mass, which is predicted, for example, by instanton models of the QCD vacuum [27]). The fourth order EChL parameters are less sensitive to the nonlocality, while the higher order ones are strongly dependent on this. Knowing the sixth order EChL parameters one can find, in principle, the corresponding effective non-local fermion action of the effective chiral quark model.

B. Polynomial contributions to the $\pi\pi$ and πK low–energy scattering paprameters

Now one can estimate polynomial contributions to the $\pi\pi$ and πK scattering parameters due to the sixth order EChL. To this end one uses values of the parameters given by eq. (5.1) and formulae (A.6–A.16) from Appendix A. The result is the following:

$\pi\pi$ scattering lengths and slope parameters

$$a_0^0 = \frac{7mB_0}{16\pi F_0^2}\{1 + \frac{4mB_0}{7m_\rho^2}(5\ln 2 + 48\frac{m_\rho^2}{F_0^2}(2L_8 - L_5)) + \frac{4m^2 B_0^2 \pi^2 h_1}{7m_\rho^4} + O(m^4)\}, \quad (5.13)$$

$$a_0^2 = -\frac{mB_0}{8\pi F_0^2}\{1 - \frac{4mB_0\ln 2}{m_\rho^2} + \frac{m^2 B_0^2 \pi^2 h_2}{m_\rho^4} + O(m^4)\}, \quad (5.14)$$

$$a_1^1 = \frac{1}{24\pi F_0^2}\{1 + \frac{8mB_0\ln 2}{m_\rho^2} + \frac{m^2 B_0^2 \pi^2 h_3}{m_\rho^4} + O(m^4)\}, \quad (5.15)$$

$$a_2^0 = \frac{\ln 2}{20\pi F_0^2 m_\rho^2} + \frac{mB_0}{240\pi F_0^2 m_\rho^4}(-\pi^2 + 72\ln^2 2 - \pi^2\frac{m_{K^*}^2 - m_\rho^2}{M_K^2}) + O(m^2), \quad (5.16)$$

$$a_2^2 = -\frac{mB_0\pi}{120F_0^2 m_\rho^4} + O(m^2), \quad (5.17)$$

$$a_3^1 = \frac{\pi^2 + 12\ln^2 2}{840\pi F_0^2 m_\rho^4} + O(m), \quad (5.18)$$

$$b_0^0 = \frac{1}{4\pi F_0^2}\{1 + \frac{4mB_0\ln 2}{m_\rho^2} + \frac{m^2 B_0^2 \pi^2 h_4}{m_\rho^4} + O(m^3)\}, \quad (5.19)$$

$$b_0^2 = -\frac{1}{8\pi F_0^2}\{1 - \frac{8mB_0\ln 2}{m_\rho^2} + \frac{m^2 B_0^2 \pi^2 h_5}{m_\rho^4} + O(m^3)\}, \quad (5.20)$$

$$b_1^1 = \frac{\ln 2}{6\pi F_0^2 m_\rho^2} + \frac{mB_0}{18\pi F_0^2 m_\rho^4}(\pi^2 + 18\ln^2 2 - \pi^2\frac{m_{K^*}^2 - m_\rho^2}{4M_K^2}) + O(m^2), \tag{5.21}$$

$$b_2^0 = -\frac{\pi}{120 F_0^2 m_\rho^4} + O(m), \tag{5.22}$$

$$b_2^2 = \frac{-\pi^2 + 18\ln^2 2}{120\pi F_0^2 m_\rho^4} + O(m), \tag{5.23}$$

where h_i are numbers not fixed by dual resonance models, they can be extracted from the analysis of masses and decay coupling constants of the pseudoscalar mesons. The constants h_i are not independent, they are related to each other by the following relations:

$$h_3 - h_4 = -\frac{17}{3} - \frac{m_{K^*}^2 - m_\rho^2}{6M_K^2} \approx -5.8, \tag{5.24}$$

$$h_4 - h_5 = 7 - \frac{m_{K^*}^2 - m_\rho^2}{M_K^2} \approx 6.2, \tag{5.25}$$

From these equations we see that numerical values of h_i can be quite large (~ 10), hence we shall use a value $h_i \approx \pm 10$ to estimate the contributions of the sixth order EChL to the $\pi\pi$ scattering parameters.

One can also make an order-of-magnitude estimate of the constants h_i by "natural" extension of the dual $\pi\pi$ amplitude, eq. (4.9), with substitutions:

$$(1 - \frac{2mB_0 \ln 2}{m_\rho^2}(1 + \frac{(2\alpha_\rho(0) - 1)m_\rho^2}{M_\pi^2}) + a_2 m^2 + \ldots) \to \frac{\sqrt{\pi}\Gamma(1 + \frac{M_\pi^2}{m_\rho^2}x)}{\Gamma(\frac{1}{2} + \frac{M_\pi^2}{m_\rho^2}x)}, \tag{5.26}$$

where $x = \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2}$. By this substitution we fix parameters a_2 and b_3 and hence h_i; the corresponding results are summarized in Table I. We see that h_i can be rather large which is in agreement with eq. (5.24,5.25). It is worth noting that this extension of the dual $\pi\pi$ amplitude, eq. (4.9), (generally speaking arbitrary) can give us only order-of-magnitude estimate of the low–energy constants but one can use this estimate in qualitative considerations.

Now one can estimate numerically the polynomial contributions to the $\pi\pi$ scattering parameters, eqs. (5.13-5.23), arising from the sixth order EChL, the corresponding numbers are given in Table II (in units of M_{π^+}). In the same table we give an experimental values of the scattering parameters taken from ref. [41], though the comparison with an

experiment is not informative before loop correction (non-analytical part of the chiral corrections) added to these quantities. From these numerical estimate we see that contributions arising from the sixth order EChL could be, in principle, as large as the fourth order ones due to the possibly large values h_i. Using eqs. (5.24,5.25) one can calculate the following combinations of the scattering length and slope parameters:

$$6a_1^1 - b_0^0 = \frac{1}{4\pi F_0^2}\left\{\frac{4mB_0\ln 2}{m_\rho^2} + \frac{m^2 B_0^2 \pi^2}{m_\rho^4}(h_3 - h_4)\right\} \approx 0.201 \cdot \left(0.0458 - 0.014\right), \quad (5.27)$$

$$2b_2^0 + b_0^0 = \frac{1}{4\pi F_0^2}\left\{\frac{12mB_0\ln 2}{m_\rho^2} + \frac{m^2 B_0^2 \pi^2}{m_\rho^4}(h_4 - h_5)\right\} \approx 0.201 \cdot \left(0.137 + 0.022\right), \quad (5.28)$$

and indeed we see that the sixth order contributions could be as large as $10\% \div 25\%$ of the fourth order ones.

The result for the low–energy parameters of the πK scattering amplitude (with $m_u = m_d = 0$) extracted from the dual resonance model eq. (4.13) is the following:

<u>πK parameters</u>

$$t_{00}^+ = 0 \qquad (\text{ exactly})$$

$$t_{01}^+ = \frac{1}{4F_0^2} + O(m_s^3)$$

$$t_{02}^+ = \frac{\ln 2}{8m_\rho^2 F_0^2} + \frac{m_s B_0 \pi^2}{48m_\rho^4 F_0^2} \cdot \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2} + O(m_s^2),$$

$$t_{03}^+ = \frac{7\pi^2 + 12\ln^2 2}{384F_0^2 m_\rho^4} + O(m_s),$$

$$t_{20}^+ = \frac{\ln 2}{8m_\rho^2 F_0^2} - \frac{m_s B_0 \pi^2}{96m_\rho^4 F_0^2} \cdot \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2} + O(m_s^2), \qquad (5.29)$$

$$t_{21}^+ = \frac{-\pi^2 + 12\ln^2 2}{128F_0^2 m_\rho^4} + O(m_s),$$

$$t_{10}^- = \frac{1}{4F_0^2} \qquad (\text{exactly!})$$

$$t_{11}^- = \frac{\ln 2}{4m_\rho^2 F_0^2} + \frac{m_s B_0 \pi^2}{96m_\rho^4 F_0^2} \cdot \frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{M_K^2} + O(m_s^2),$$

$$t_{12}^- = \frac{\pi^2 + 12\ln^2 2}{128F_0^2 m_\rho^4} + O(m_s),$$

$$t_{30}^- = \frac{\pi^2 + 12\ln^2 2}{384F_0^2 m_\rho^4} + O(m_s).$$

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From these expressions we see that in the general dual (string) model compatible with soft–pions theorem the explicit symmetry breaking parameter is not M_K^2/m_ρ^2 but rather $\frac{m_{K^*}^2 - m_\rho^2 - M_K^2}{m_\rho^2}$. The latter parameter being of order $\sim m_s \sim M_K^2/m_\rho^2$ has an additional numerical suppression.

For the πK scattering parameters the contributions of the sixth order EChL are fixed unambiguously by *exact* low–energy theorems eqs. (3.22,3.29). Substituting numerical values of the parameters eq. (5.6) into the eqs. (5.29) one gets results showed in Table III (in units of M_{π^+}). In this table we also show, for completeness, the experimental values of the low–energy parameters obtained by Lang and Porod [42]. Let us stress again that to compare chiral results with experimental data one needs to add to the tree–level results (showed in Table III) the loop corrections.

Surprisingly, the contributions of polynomial part of the sixth order corrections are rather small (less than 10%). Moreover these corrections are exactly zero if one impose the Adler conditions eq. (4.10). The smallness of the polynomial part of the sixth order contribution to the πK amplitude has been discussed in ref. [44]. It has been shown on general grounds that the low–energy theorems for the πK scattering are technically respected through the cancellation of different resonances contributions to πK scattering at low-energies. Say, for the parameter t_{20}^+ this cancellations is not exact (like for t_{00}^+ and t_{10}^-) but nevertheless, even being partial it leads to a relative smallness of the strange quark mass corrections to this parameter [44].

VI. SUMMARY AND CONCLUSIONS

To summarize, we have calculated parameters of the sixth order effective chiral lagrangian in the large N_c limit from the dual resonance (string) model for the scattering amplitudes of the (pseudo)Goldstone particles. The results are summarized in Table I. These parameters determine the polynomial terms in the low–energy expansion of the scattering amplitudes up to the order $O(p^6)$. The polynomial contributions being com-

bined with non–analytical parts of the amplitudes arising from meson loops would enable us to make a precise calculation of the sixth order contributions to the low–energy scattering parameters. From our analysis of the polynomial part of the sixth order corrections one can conclude that those corrections to the $\pi\pi$ scattering parameters can be as large as $10 \div 25\%$ of the fourth order ones, whereas the analogous corrections to πK low-energy scattering parameters are surprisingly small (usually less than 10%, in spite of naive expectation of $M_K^2/m_\rho^2 \sim 40\%$). The smallness is explained by "accidentally" small parameter $\frac{m_{K*}^2 - m_\rho^2 - M_K^2}{m_\rho^2} \sim 7\%$ which plays a role of explicit chiral symmetry breaking parameter in the dual resonance (string) models.

Apart from the "practical" value, our studies may have wider theoretical significance. We found that a dual resonance model with "built in" soft–pion theorems is consistent with the non–topological chiral anomaly of the QCD what might be an indication of the deep relations between QCD and some string theory. On other side, application of the soft–pion theorems to dual resonance models leads to the prediction of the universality of the ρ- and K^*- Regge trajectories slopes. Comparing the predictions for the sixth order EChL in the effective chiral quark model with ours we found that the sixth order EChL from the dual resonance model differs from that obtained by gradient expansion of the fermion determinant in the effective chiral–quark model[6]. In our view the reason for this difference is that doing gradient expansion of the fermion determinant [39,40] to the sixth order one has to take into account a non-locality of the effective fermion action (say, the momentum dependence of the constituent quark mass). Knowing the sixth order EChL parameters one can find, in principle, the corresponding effective non-local fermion action of the effective chiral quark model. This work is in a progress.

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[6]The corresponding expansion of the fermion determinant to the fourth order reproduces the non-topological chiral anomaly results [27] and so the fourth order of the gradient expansion is consistent with dual models.

VII. ACKNOLEGEMENTS

Appendix.

Projecting out amplitudes of definite isospin in s-channel yields:

$$M^0(s,t) = 3A(s,t,u) + A(t,u,s) + A(u,s,t), \tag{A.1}$$

$$M^1(s,t) = A(t,u,s) - A(u,s,t), \tag{A.2}$$

$$M^2(s,t) = A(t,u,s) + A(u,s,t), \tag{A.3}$$

where $A(s,t,u)$ is defined by eqs. (3.5,3.7). In the center of mass frame:

$$s = 4(q^2 + M_\pi^2),$$

$$t = -2q^2(1 - \cos\theta),$$

$$u = -2q^2(1 + \cos\theta), \tag{A.4}$$

where q is the spatial momentum and θ is the scattering angle. We then define the partial wave isospin amplitudes according to the following formula:

$$M^I(s,t) = 32\pi \sum_{l=0}^{\infty}(2l + 1)P_l(\cos\theta)M_l^I(s)$$

The behaviour of the partial waves near threshold is of the form

$$\text{Re } M_l^I(s) = q^{2l}\{a_l^I + q^2 b_l^I + O(q^4)\} \tag{A.5}$$

The quantities a_l^I are referred to as the $\pi\pi$ scattering lengths and b_l^I as slope parameters. They can be expressed in terms of the low–energy subthreshold expansion parameters A_{kl} defined by eq. (3.10) as follows:

$$a_0^0 = \frac{1}{32\pi}\{5A_{00} + 12M_\pi^2 A_{10} + 48M_\pi^4 A_{20} + 192M_\pi^6 A_{30} + O(M_\pi^8)\}, \tag{A.6}$$

$$a_0^2 = \frac{1}{16\pi}A_{00} + O(M_\pi^8), \tag{A.7}$$

$$a_1^1 = \frac{1}{24\pi}\{A_{10} + 4M_\pi^2 A_{11} + 16M_\pi^4 A_{12} + O(M_\pi^6)\}, \tag{A.8}$$

$$a_2^0 = \frac{1}{60\pi}\{3A_{11} + 2A_{20} + 32M_\pi^2 A_{12} + O(M_\pi^4)\}, \tag{A.9}$$

$$a_2^2 = \frac{1}{30\pi}\{A_{20} + 4M_\pi^2 A_{12} + O(M_\pi^4)\}, \tag{A.10}$$

$$a_3^1 = \frac{1}{35\pi}A_{30} + O(M_\pi^2), \tag{A.11}$$

$$b_0^0 = \frac{1}{4\pi}\{A_{10} + 2M_\pi^2(A_{11} + 6A_{20}) + 8M_\pi^4(A_{12} + 9A_{30}) + O(M_\pi^6)\}, \tag{A.12}$$

$$b_0^2 = -\frac{1}{8\pi}\{A_{10} - 2M_\pi^2 A_{11} - 16M_\pi^4 A_{12} + O(M_\pi^6)\}, \tag{A.13}$$

$$b_1^1 = \frac{1}{6\pi}\{A_{11} - A_{20} + 4M_\pi^2 A_{12} + O(M_\pi^4)\}, \tag{A.14}$$

$$b_2^0 = \frac{1}{15\pi}\{5A_{12} - 3A_{30} + O(M_\pi^2)\}, \tag{A.15}$$

$$b_2^2 = \frac{1}{15\pi}\{2A_{12} - 3A_{30} + O(M_\pi^2)\}, \tag{A.16}$$

where we take into account the Bose symmetry requirements :

$$A_{21} = A_{12}, \tag{A.17}$$

$$A_{01} = -4M_\pi^2 A_{02}, \tag{A.18}$$

$$A_{02} = A_{11} + 4M_\pi^2 A_{21}. \tag{A.19}$$

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TABLES

	DRM with $M_{K^*} = 872$ MeV	DRM with Adler conditions	" extended " DRM
K_1	0	0	0
K_2	-3.72	-3.72	-3.72
K_3	1.61	1.61	1.61
K_4	-1.34	-1.34	-1.34
K_5	1.61	1.61	1.61
$\tilde{K}_6$	-1.26	-1.61	-1.26
$\tilde{K}_7$	2.78	2.66	2.78
$\tilde{K}_8$	2.07	2.42	2.07
$K_9 - K_{12}$	1.03	1.21	1.03
$K_{10} - K_{12}$	0.21	0.21	0.21
$K_{11} - K_9$	–	–	-2.74
h_1	–	–	4.02
h_2	–	–	0.48
h_3	–	–	1.77
h_4	–	–	7.70
h_5	–	–	1.54

TABLE I. Low energy coupling constants of the sixth order EChL eq. (2.4) K_i in units of $10^{-3}/m_\rho^2$ for different types of the dual resonance models (DRM)

	$\mathcal{L}^{(2)}$	$\mathcal{L}^{(4)}$	$\mathcal{L}^{(6)}$	$\mathcal{L}^{(6)}$ from " extended" DRM	experiment [41]
a_0^0	0.18	$(5.9 \pm 6.7) \cdot 10^{-3}$	$0.27 \cdot 10^{-3} h_1$	$1.09 \cdot 10^{-3}$	0.26 ± 0.05
$-10 \cdot a_0^2$	0.51	$-2.3 \cdot 10^{-2}$	$0.14 \cdot 10^{-2} h_2$	$0.067 \cdot 10^{-2}$	0.28 ± 0.12
$10 \cdot a_1^1$	0.33	$3.1 \cdot 10^{-2}$	$0.09 \cdot 10^{-2} h_3$	$0.16 \cdot 10^{-2}$	0.38 ± 0.02
$10^3 \cdot a_2^0$	0	0.92	0.03	0.03	1.7 ± 0.3
$10^3 \cdot a_2^2$	0	0	-0.037	-0.037	0.13 ± 0.3
$10^3 \cdot a_3^1$	0	0	0.016	0.016	0.06 ± 0.02
b_0^0	0.20	$0.93 \cdot 10^{-2}$	$0.06 \cdot 10^{-2} h_4$	$0.46 \cdot 10^{-2}$	0.25 ± 0.03
$-10 \cdot b_0^2$	1.0	$-9.3 \cdot 10^{-2}$	$0.28 \cdot 10^{-2} h_5$	$0.43 \cdot 10^{-2}$	0.82 ± 0.08
$10^2 \cdot b_1^1$	0	0.31	0.040	0.040	-
$10^5 \cdot b_2^0$	0	0	7.2	7.2	-
$10^5 \cdot b_2^2$	0	0	-0.90	-0.90	-

TABLE II. Tree level contributions to the $\pi\pi$ scattering lengths and slope parameters from the EChL at different orders. The constants of the $\mathcal{L}^{(6)}$ are extracted from dual resonance models as explained in the text (see also Table I)

	$\mathcal{L}^{(2)}$	$\mathcal{L}^{(4)}$	$\mathcal{L}^{(6)}$	data from [43]	all data
t_{00}^+	0	0	0	1.31 ± 1.26	0.52 ± 2.03
t_{01}^+	0.63	0	0	0.75 ± 0.09	0.55 ± 0.07
$10^3 \cdot t_{02}^+$	0	7.2	-1.23	-	-
$10^3 \cdot t_{03}^+$	0	0	0.54	-	-
$10^3 \cdot t_{20}^+$	0	7.2	0.62	17.7 ± 1.3	10.3 ± 1.0
$10^3 \cdot t_{21}^+$	0	0	0.089	(0.9 ± 0.15)	(0.2 ± 0.1)
t_{10}^-	0.63	0	0	1.00 ± 0.07	0.52 ± 0.07
$10^3 \cdot t_{11}^-$	0	14.5	-0.61	22 ± 5	14.9 ± 2.8
$10^3 \cdot t_{12}^-$	0	0	0.34	0.33 ± 0.01	0.55 ± 0.1
$10^3 \cdot 3t_{30}^-$	0	0	0.34	-	-

TABLE III.   Tree level contributions to the low-energy parameters (defined by eqs. (3.11, 3.12)) of the πK scattering amplitude from the EChL at different orders. The constants of the $\mathcal{L}^{(6)}$ are extracted from dual resonance models as explained in the text (see also Table I). The experimental values are the results of dispersion-theoretical analysis of Lang and Porod [42]